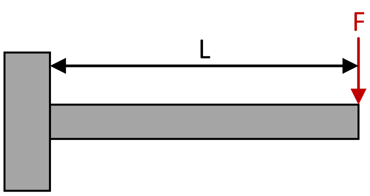
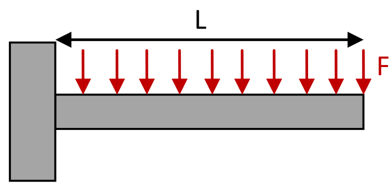
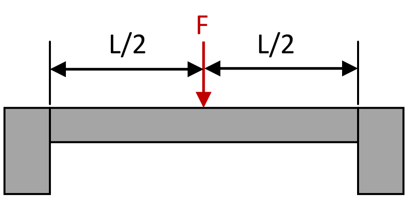
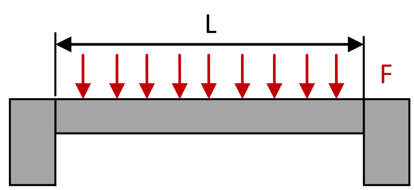
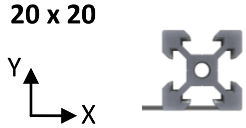
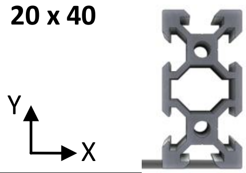
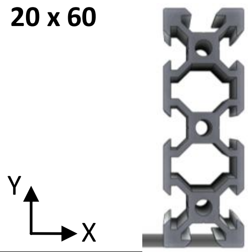
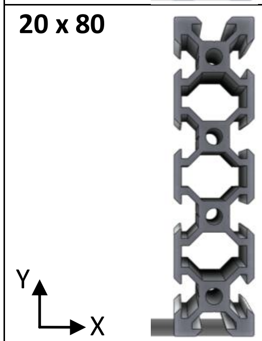
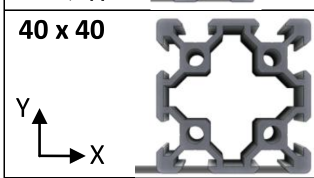
How to Estimate Deflection in Your Open Builds Projects

While designing buildings and structures, engineers calculate the amount a beam will bend, or deflect under load. The same formulas can be used to estimate how much a V-slot profile will bend, which could be an important consideration for 3D printers and other builds. The diagrams below illustrate common beam support and load conditions, along with deflection formulas. The parameters are as follows

F: force acting on the beam
L: length of the beam between supports
E: modulus of elasticity (a material property representing stiffness)
I: moment of inertia (a value dependant on the cross sectional shape of the beam)

Cantilever beam with a concentrated load  Maximum deflection = $\frac{FL^3}{3EI}$	**Cantilever beam with a distributed load**  Maximum deflection = $\frac{FL^3}{8EI}$ F is the total force distributed evenly across the length
Simply supported beam with a concentrated load  Maximum deflection = $\frac{FL^3}{48EI}$	**Simply supported beam with a distributed load**  Maximum deflection = $\frac{5FL^3}{348EI}$ F is the total force distributed evenly across the length

The moment of inertia (I) is a measure of how resistant to bending a particular beam cross section will be. The greater the area and the further away the area is from the bending axis, the higher the moment of inertia will be. A square profile has the same moment of inertia in both x and y axes, but notice that the rectangular profiles have a much higher moment of inertia in the taller orientation.

Profile	Area Moment of Inertia [m^4]	
20 x 20 	**Ix** 6.988×10^{-9}	**Iy** 6988.35
20 x 40 	**Ix** 48.163×10^{-9}	**Iy** 12.305×10^{-9}
20 x 60 	**Ix** 149.336×10^{-9}	**Iy** 17.622×10^{-9}
20 x 80 	**Ix** 336.224×10^{-9}	**Iy** 22.938×10^{-9}
40 x 40 	**Ix** 81.407×10^{-9}	**Iy** 81.407×10^{-9}
C-Beam 	**Ix** 117.760×10^{-9}	**Iy** 565.646×10^{-9}

Sample Calculation

Predict the deflection caused by a single 10 kg load on a 1000mm length of 20 x 40 V-slot supported at both ends.

1) The force is calculated by multiplying the mass by the acceleration due to gravity (9.81 m/s^2).

$$F = ma$$
$$F = 10 \; x \; 9.81$$

$$F = 98.1\ N$$

2) The length of the beam is 1000mm, but the formula requires base units of m,kgP,N,Pa, etc, so the length we will use is 1m.

3) The particular Aluminum alloy V-slot profiles are made from is 6063-T5. The modulus of elasticity, E for this material is 68.9 GPa (gigapascals). Once again, this needs to be in base units, so it is $68.9 x 10^9$ Pa.

3) The moment of inertia about the x axis for 20 x 40 V-slot is found in the table above: $48.163\ x 10^{-9}\ m^4$.

4) The applicable deflection formula for this load condition is found in the first table:

$$\frac{FL^3}{48EI}$$

$$= \frac{98.1\ x\ 1^3}{48\ x\ 68.9x10^9\ x\ 48.163x10^{-9}}$$

$$= 0.000615\ m\ , or\ 0.615\ mm$$

Notes

A common mistake when using these formulas is entering values with incorrect units. Everything needs to be in base units. Force must be in Newtons, length must be in meters, the modulus of elasticity must be in Pascals, and the moment of inertia must be in $meters^4$.

Keep in mind that these calculations assume ideal support conditions, and in reality things are often over constrained, so the deflection will be different. It can also be difficult to predict the real loading conditions. So the deflection values you get will not be exact, but they are helpful as a design guide.

The 10^9 and 10^{-9} cancel each other out, so you can leave them out of the calculation.

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